

Head-To-Head Tie-Breaker

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September 30, 2015

1. What is the Head-To-Head Tie-Breaker?

When sports teams compete in a league, the athletes on those teams are usually very interested in the standings. They want to know how their team compares to the other teams in the competition.

The most common method of ranking teams in leagues is Winning Percentage. Ignoring ties, Winning Percentage is calculated by dividing the number of games a team has won by the total number of games played. Because it is fairly common for two teams to be tied with the same Winning Percentage, most leagues have a sequence of additional criterion that can be applied to break those ties and uniquely rank the teams. The Head-To-Head (HTH) criterion is the most common first tie-breaker, which means the HTH criterion is applied to any teams with identical Winning Percentages.

The HTH tie-breaker is very simple when only two teams are tied, but the HTH analysis becomes much more complex when applied to three or more teams. The QuickScores website currently supports 16 different ways of ranking teams in league standings, but when a client calls for technical support relating to team standings, by far the most common questions concern the HTH tie-breaker when three or more teams are tied.

The purpose of this paper is to explain how the Head-To-Head tie-breaker is calculated and applied in QuickScores.

2. Possible Outcomes From Applying The Head-To-Head Criterion

Although the goal of the HTH algorithm is to break a tie and uniquely rank each team, sometimes not enough games have been played between the tied teams to draw any conclusions. The simplest case is when two teams are tied, but they haven't yet played each other. In cases involving three or more teams, there may have been games played between some of the tied teams, but not enough games to make any logical conclusions. When no conclusions can be drawn, QuickScores will display "--" in the HTH column of the league standings, and the software will move on to apply the next tie-breaker.

If enough games have been played to draw conclusions, the software will display a number in the HTH column of the league standings. The best team will have the number 1, the second best team will have a 2, and so on. Sometimes multiple teams will have the same number.

Sometimes two teams are tied and they have played each other, but the teams tied in the one game they played, or they played two games and each team won once. In this case, the two teams will remain tied, be listed with the same HTH number in the standings, and the software will move on to the next tie-breaker.

If three or more teams are tied and their game record represents a contradiction (this will be explained below), all of the teams will have a 1 in the standings and the next tie-breaker will be applied.

3. Logical Deduction

The whole concept of maintaining standings and ranking teams assumes that a team's quality is something that can be measured and used to compare with the quality of other teams. The team with the most quality is ranked #1.

We are all accustomed to measuring and ranking people and things. If we wanted to find the heaviest person in a group, we could have one person at a time stand on a bathroom scale, write down their weights, and then rank them from heaviest to lightest. Similarly, if you wanted to find the tallest person, you could use a tape measure to come up with an absolute height for each person.

Sometimes you want to rank things according to a characteristic that can only be measure *relatively*, rather than *absolutely*. For example, if you want to rank by weight, but you don't have a bathroom scale, you could put a person on either end of a seesaw. Whoever is down is heavier than the person in the air. Similarly, without a tape measure, you can still rank people by height by pairing them up and having them stand back-to-back.

Ranking sports teams is accomplished using *relative* measurements. Two teams play a game against each other, and you record whether the teams tied or one of the teams won. With enough of these relative measurements of quality, you hope that you can unambiguously determine the relative rankings of the teams.

When dealing with physical measurements, like height or weight, people are comfortable using logical deductions. For example, if we know that person A is taller than person B, and that person B is taller than person C, we have no problem concluding that person A is taller than person C, even though A and C had never stood back-to-back for comparison. The Head-To-Head algorithm in QuickScores uses similar logical deductions. If we know that team A is better than team B (because they played each other and A won), and we similarly know that team B is better than team C, then we conclude that team A is better than team C even though they may not have played each other yet.

Occasionally an athlete will object to this type of deduction and cite the NFL mantra: "On any given Sunday, any team can beat any other team." Even if that were literally true and the results of any game were as random as flipping a coin, the standings must be based on the games that have actually been played. The standings cannot be based on what might possibly happen in the future. If and when A plays C in the future, the results of that game will be figured into the standings appropriately.

4. Reduction to a Single Relative Comparison

The first step in the QuickScores HTH algorithm is to compare each pair of teams that are tied, consider the results of all the games (if any) those teams have played against each other, and either declare the two teams to be equal, or declare one team is better than the other.

If three teams – A, B, and C - are tied, then you must compare three pairs: A-B, B-C, and A-C. For each pair, consider the wins, losses, and ties between only those two teams and determine if they are equal or one is better. For example, if A and B have played once and A won that game, then A is considered better than B. If A and B played three times, and A won two of the three, you again conclude that A is better than B. If A and B played five times and A won all of the games, the conclusion is simply that A is better than B.

If one team dominates another team over multiple games, it does not mean that the dominant team is much much better than the weaker team. All it means is that the dominant team is *consistently better*, but it gives no indication of *how much* better that team is. (The Head-To-Group variation erroneously assumes that multiple victories over the same team somehow elevates a team's quality over other teams.)

If two teams play one time and their score is tied, or if two teams play twice and each team wins once, the two teams are considered equal.

5. Simple Graphical Notation

When analyzing the HTH tie-breaker in leagues with three or more teams tied, there is a simple graphical notation on paper that makes it easier to understand the situation.

Identify the teams that are tied and write the names widely spaced apart on a sheet of paper. Then read through the league schedule and use lines to connect the names to indicate the results of a game that has been played.

If a game ends with a definite winner, draw an arrow from the winner to the loser. Figure 1 to the right shows that Team A beat Team D. If two teams have played multiple times, draw as many arrows as needed.

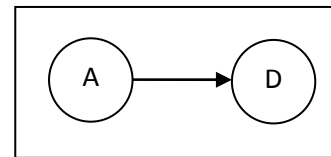


Figure 1 - Team A beat Team D

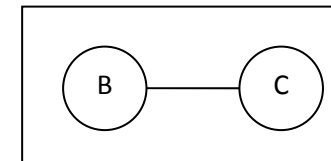


Figure 2 - Team B equals Team C

If two teams play and the game ends in a tie, draw a line between the two circles without an arrowhead, such as between B and C in Figure 2.

After recording every game as either an arrow or a straight line, look at every pair of teams with multiple connections and determine whether one team is better than the other, or the two teams are tied. When you have made that determination, erase all of the connections between those two teams and add a single arrow or a straight line between the teams. In the end, there should only be a single line or arrow between any two teams.

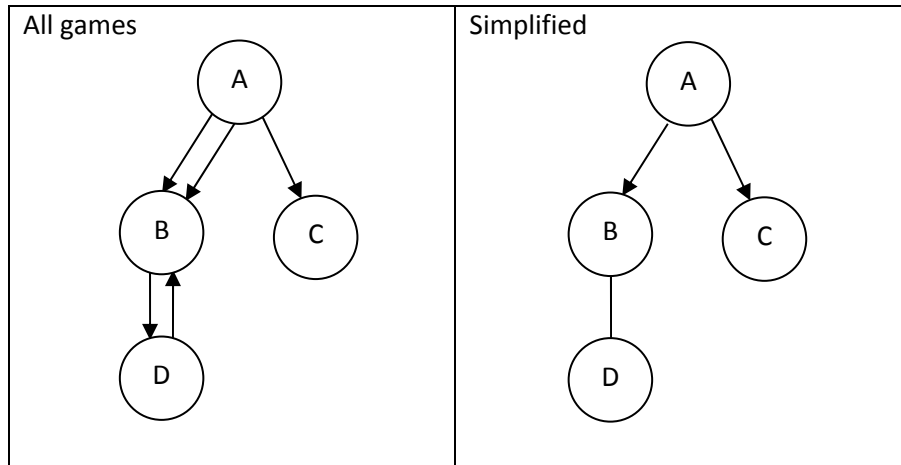


Figure 3 - Reducing a diagram to its simplest form

The two diagrams above show how the HTH games should be simplified. The left diagram shows the five games played between the four tied teams. The right diagram shows how the two victories that A had over B is reduced to a single arrow, and the two games between B and D are reduced to a single equality line. If there is only one line between two teams, such as between A and C, no reduction is necessary.

6. Partial Resolution

Sometimes there are enough games between the tied teams to draw some conclusions, but not enough games to fully rank all of the teams. The two examples on the right show the most common situations.

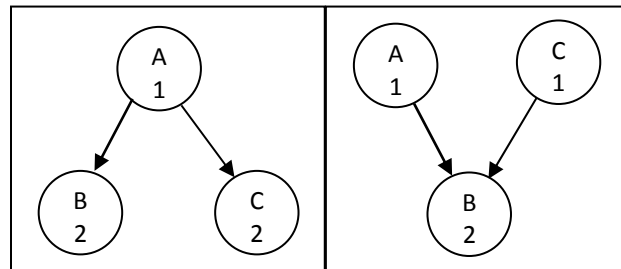


Figure 4 - Partial Resolution

In the left example, A has beaten both teams B and C, but B and C have not yet played each other. In this case, A is clearly the best team and would be given a 1 in the HTH column of the standings. There is no way to determine the relative ranking of teams B and C, so both teams would be given a 2 in the HTH column of the standings and the software would move on to the next tie-breaker. The next tie-breaker criterion would only be applied to B and C.

Using similar reasoning on the right-hand example, Team B would be given a 2 in the HTH column of the standings, while teams A and C would both be given a 1. The next tie-breaker criterion would be used to break the continuing tie between A and C.

7. Ambiguity

When only two teams are tied and those two teams haven't played each other, everyone understands that the HTH criterion cannot be applied. However, there is a common related situation, depicted to the right, which many people misunderstand.

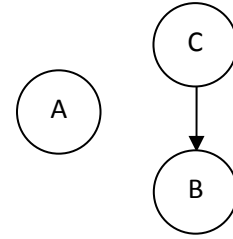


Figure 5 - Ambiguity

As shown in Figure 5, assume teams A, B, and C are tied with the same winning percentage. C has beaten B, but A has not played either B or C. Based on the subsequent tie-breakers, the final standings may show Team C ranked below Team B, and that often prompts the captain of Team C to complain. "We can't be below Team B, because we beat them. The head-to-head tie-breaker should put us on top."

That is a tantalizing argument, especially when the previous section of this paper describes the possibility of a partial resolution. How is this different from the previous examples? Why can't we partially resolve that C is better than B even though we don't know where A fits in?

Let's go back and more closely analyze the left-hand example of Figure 4 on the previous page, where A has beaten both B and C, but B and C haven't played. There are three possible true relationships between the three teams.

$A > B > C$ $A > B = C$ $A > C > B$

In all three cases, Team A is on the left, so we can conclude that A is the best team even though we don't know the relationship between B and C.

Looking at Figure 5 on this page, where C beat B, but A hasn't played either team, there are five possible true relationships between the three teams.

$A > C > B$ $A = C > B$ $C > A > B$ $C > A = B$ $C > B > A$

There is no consistency in these five cases. No team is always on the left, or in the middle, or on the right. No conclusions are possible, partial or otherwise. In this case, QuickScores will give all three teams a 1 in the HTH column of the standings and move on to the next tie-breaker criterion to break the continuing three-way tie. To the consternation of Team C, the next tie-breaker may not rank C higher than B.

8. Contradictions

When three teams are tied, it is not uncommon to have the situation depicted on the right. Team A beat Team B, Team B beat Team C, and Team C beat Team A.

In this circular case, reminiscent of rock-paper-scissors, there is no way to declare that any team is better or worse than any other team. QuickScores recognizes this circular contradictory situation, gives all three teams the same HTH number, and moves on to the next tie-breaker criterion.

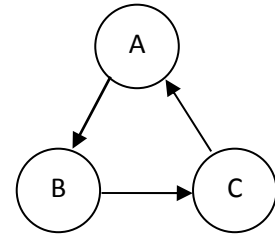


Figure 6 - Circular contradiction

9. Variations on the standard HTH algorithm

We at QuickScores strongly recommend that our clients use our default HTH algorithm that uses logical deduction as described above. However, over the years we have learned that a small minority of our clients prefer to use various other HTH algorithms. Although we can point out deficiencies in these alternatives, we are not able to beat “That’s the way we’ve always done it” or “That’s the way it’s written in our rules.” So in the interest of providing our clients with the service they want, we offer the following three variations to the HTH algorithm.

9.1 Variation #1 – Head-To-Head (two only)

As described in the Introduction, everyone understands and agrees on the HTH results when only two teams are tied. Questions and arguments only arise when three or more teams are tied. To avoid these issues, a few organizations declare that the Head-To-Head tie-breaker will only be applied if two teams are tied. If three or more teams are tied, the HTH tie-breaker is ignored and the software moves on to consider the next tie-breaker.

9.1.1 Issues with Head-To-Head (two only)

While this HTH variation may avoid some arguments, it may cause other arguments because it fails to rank teams in some very clear cases.

In the example in Figure 7, team A beat teams B and C, and team B beat C. Virtually everyone would agree that A is the best, B is second best, and C is worst. But the Head-To-Head (two only) variation would refuse to rank these teams because there are more than two teams involved.

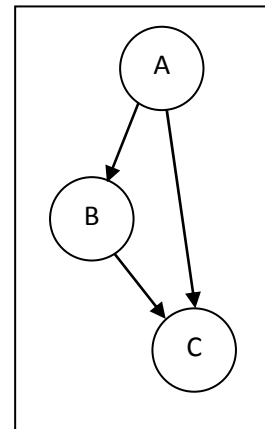


Figure 7 – Unambiguous, but ignored by HTH (two only)

9.2 Variation #2 – Head-To-Head (all play)

Some organizations believe it is wrong to use logical deduction to predict the outcome of two teams playing a game. If team A beat team B, and B beat C, a proponent of this HTH variation would contend that you should not jump to the conclusion that team A is better than team C.

Rather than viewing team quality as a physical measurement like height or weight that can be arithmetically compared, proponents view games as much more complex interactions between the unique characteristics of each team pairing. (“We have the best curveball pitchers in the league, and that team has the worst curveball hitters,” or “We knocked them out of the playoffs the last three years, so we have them psyched out”.)

In this HTH variation, since logical deduction is forbidden, teams will only be ranked if all tied teams have played all of the other tied teams. If any pair of teams has not played, the HTH tie-breaker is ignored and the software moves on to consider the next tie-breaker.

9.2.1 Issues with Head-To-Head (all play)

First, while it is possible that a weak team has some special advantage over a seemingly stronger team, that is rarely the case. Why withhold judgment on ranking teams because of an advantage that may or may not be real?

Second, standings are supposed to be a ranking of the teams based on games that have been played up to that point in time. Even if everyone in the league believes that a future game will change the standings, the standings today should not reflect anticipated future results.

Third, if standings were truly intended to be based on expected (hoped for?) future results, who is to say that two teams playing once is enough? If team A plays team B once, and A wins, is that enough to say A is better? If A and B play three times during the season and A wins the first encounter, B could claim that they might beat A the next two times they play, so it’s premature to declare A the better team after only one game.

9.3 Variation #3 – Head-To-Group

This variation takes the simple calculation that works when two teams are tied and applies it when more than two teams are tied.

When two teams are tied, all of the games are considered that were played between those two teams and the winning percentage for each team is calculated. Whichever team has the highest winning percentage is declared the better of the two teams. If the two teams have not played each other, this tie-breaker does not apply.

If three or more teams are tied (assuming every team has played at least one game), this HTH variation considers all of the games played between the tied teams, calculates the winning percentage for each team, and ranks them according to that percentage.

9.3.1 Issues with Head-To-Group

The popularity of this variation comes from its simplicity, but unfortunately there are times when this method arrives at incorrect rankings. The following are a couple examples.

Assume four teams are tied, as shown in Figure 8 below. Team A beat team B, B beat C, and C beat D. Most people would agree with our standard HTH algorithm that ranks A as 1st, B as 2nd, C as 3rd, and D as 4th. However, the Head-To-Group calculation gives both B and C identical .500 winning percentages, so it would rank B and C as equal.

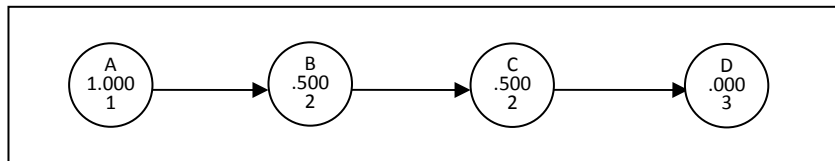


Figure 8 - Four-way tie incorrectly ranked

In the left diagram of Figure 9 below, we have the classic three-way circle of teams that form a contradiction. The Head-To-Group method calculates identical .500 winning percentages for each of the three teams, and thus correctly considers them all equal. In the right diagram below, the only change is a second game between B and C that confirms B is better than C. The circular contradiction still exists, but the Head-To-Group method erroneously declares B to be the best team because it has the highest winning percentage.

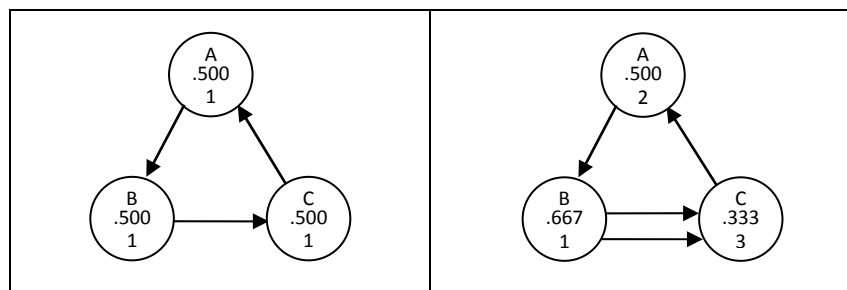


Figure 9 - Circular contradictions

10. Head-To-Head Differential Tie-Breaker

The Head-To-Head Differential tie-breaker is used much less often than the normal Head-To-Head tie-breaker. When the Differential criterion is used, it is nearly always immediately following the normal HTH criterion.

In normal HTH analysis, a win is a win; there is no consideration of whether the victor trounced the loser, or the victor just barely squeaked by. In HTH Differential, the size of the victory does matter, but only when two teams have met more than once.

In normal HTH, if two teams have played twice and each team won once, the two teams are considered equal. In HTH Differential, if two teams split games, the margins of victory are calculated and the team with the larger margin is considered the better team.

Generalizing this to any number of games (and ignoring for now the Max Differential value), consider all of the games played between teams A and B. For each team, add up the number of points or runs they scored in the games, and subtract the number of points or runs scored by the other team. Whichever team has the larger number is considered the better team in the HTH Differential tie-breaker.

Using a differential tie-breaker is generally a good thing, but one runs the risk of a strong team running up the score on a much weaker team and making the game unsporting. To counteract this tendency, QuickScores allows you to define a Max Differential value which defines an upper limit on the number of differential runs or points that can be earned in any single game. In softball, a typical Max Differential value is 10. That means no matter how big a differential there is in the real game score, only a difference of 10 will be applied towards the tie-breaker. The Max Differential value applies to the Head-To-Head Differential tie-breaker as well as the Average Run Differential tie-breaker.

Sometimes the HTH Differential analysis will come to a different conclusion than the normal HTH analysis. Suppose teams A and B have played three games, with Team A winning two games, each with a margin of one, while Team B wins one game with a margin of ten. The normal HTH analysis would consider A to be the better team due to the 2 to 1 victories. The HTH Differential analysis would consider B to be the better team due to the aggregate eight in victory margins.

It should be noted that if two teams have only played once, the normal HTH and HTH Differential will always come to the same conclusions. The differential is only significant when there are multiple games between the same two teams. In most recreational sports leagues, a team only plays the other teams once, or maybe only plays a few of the teams a second time. For this reason, the HTH Differential tie-breaker is seldom needed in recreational leagues.

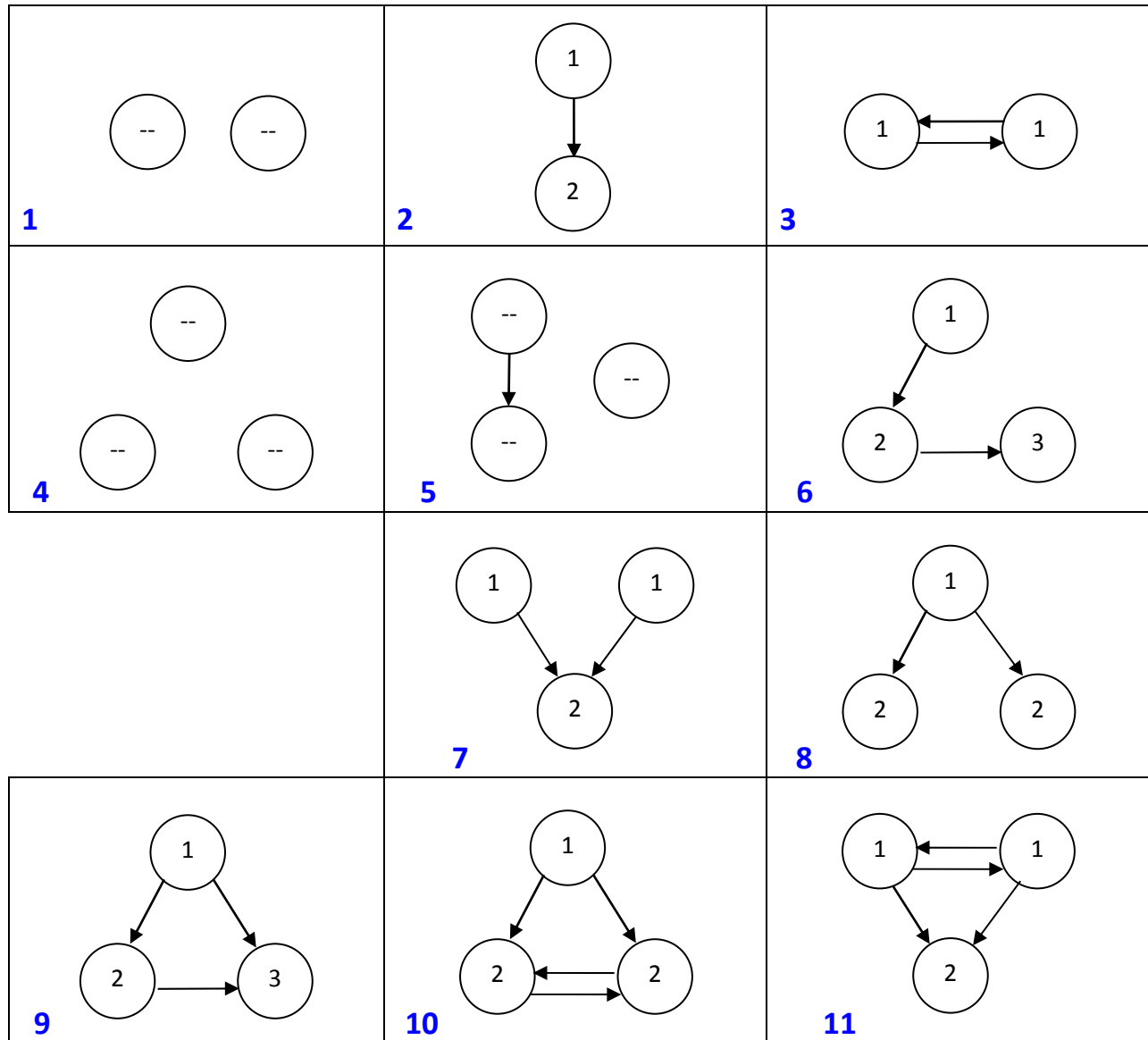
Appendix – All Head-To-Head Test Cases

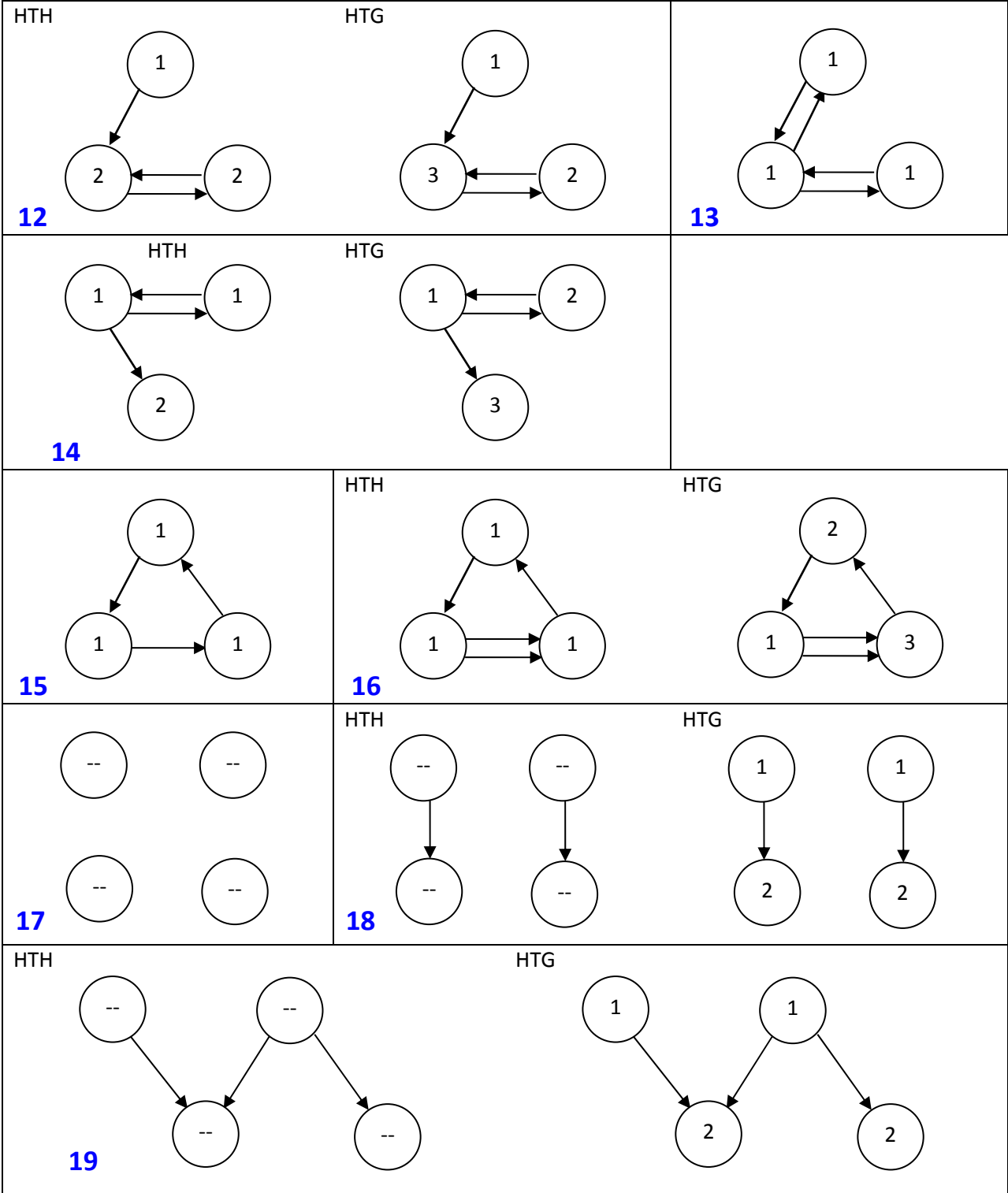
In order to test the QuickScores software, 34 different test cases were created using as many as seven teams that are tied in various configurations. The numbers in the circles indicate the ranking that team would have in the standings.

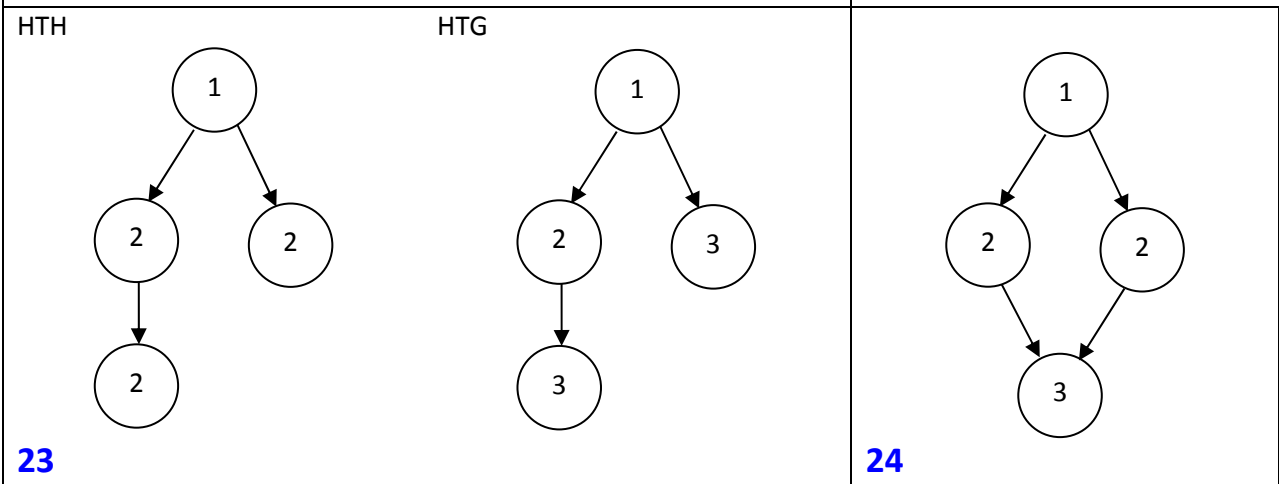
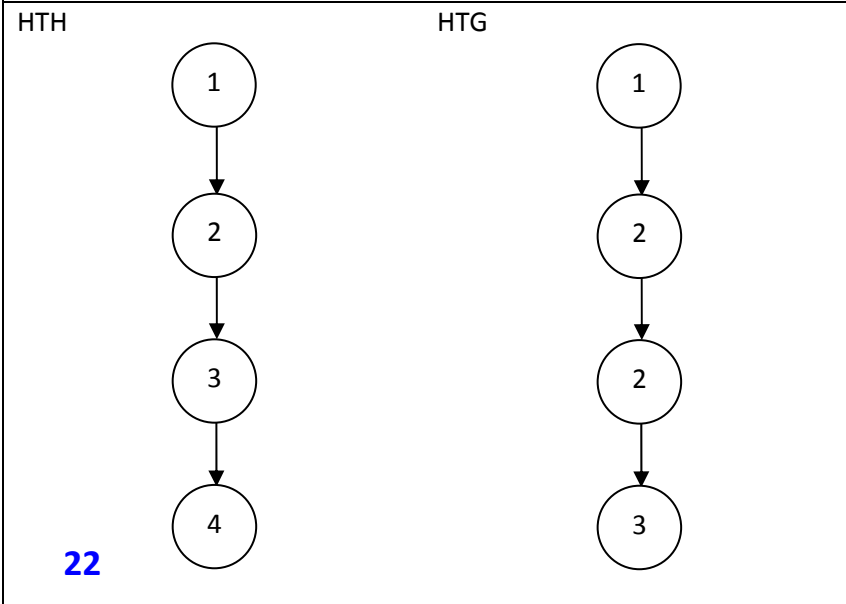
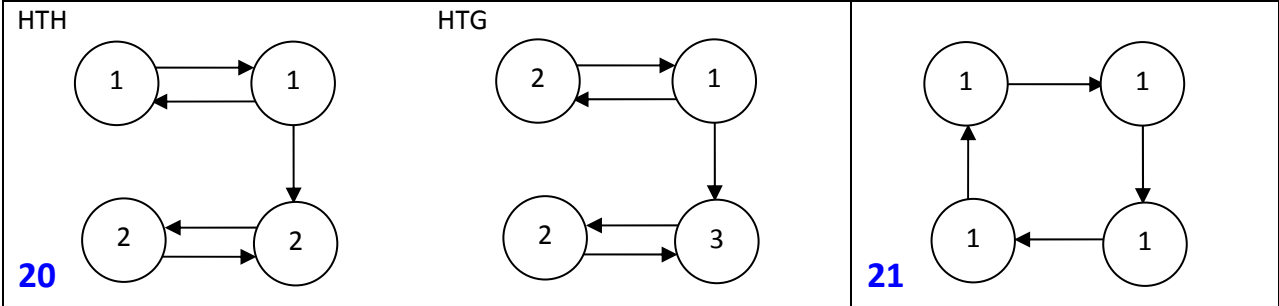
“HTH (two only)” only yields results in the first three cases, those with only two teams. In all other cases, the teams are given a non-committal “- -” value.

“HTH (all play)” only yields results when all teams have played all other teams. In all other cases, the teams are given a non-committal “- -” value.

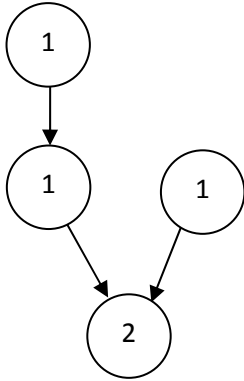
In those cases where the standard Head-To-Head method yields different results from the Head-To-Group method, there is a notation in the upper left corner of the case.





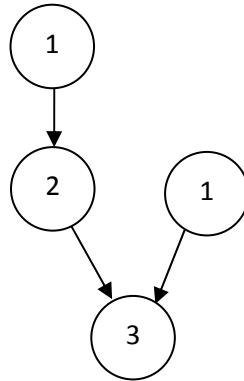


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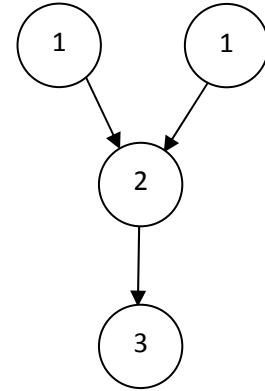


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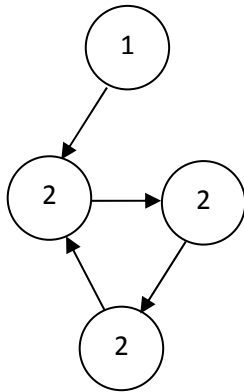
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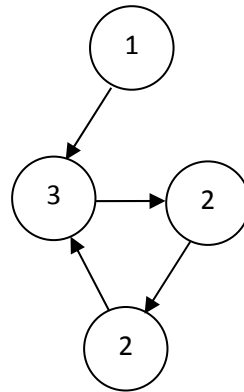


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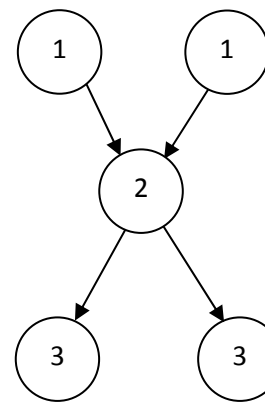


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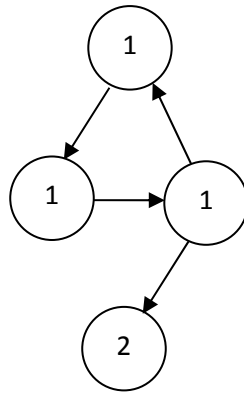
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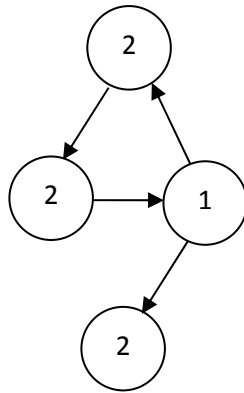


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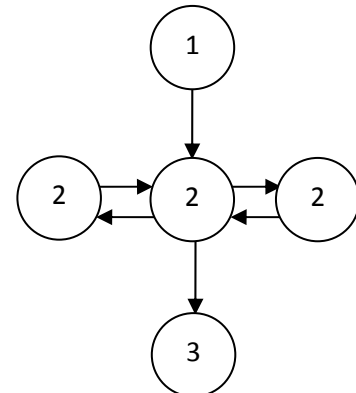


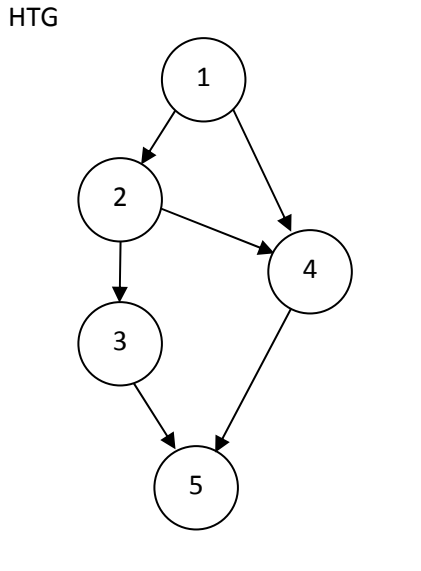
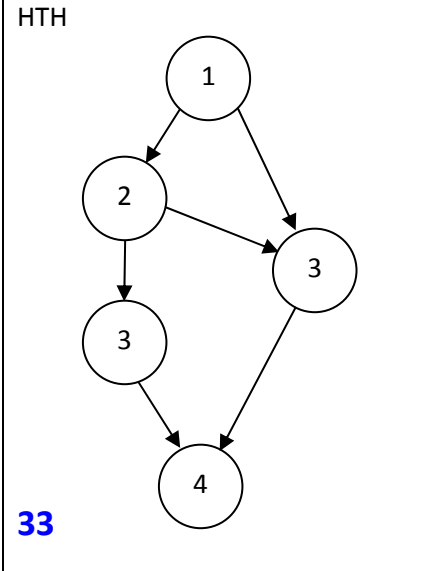
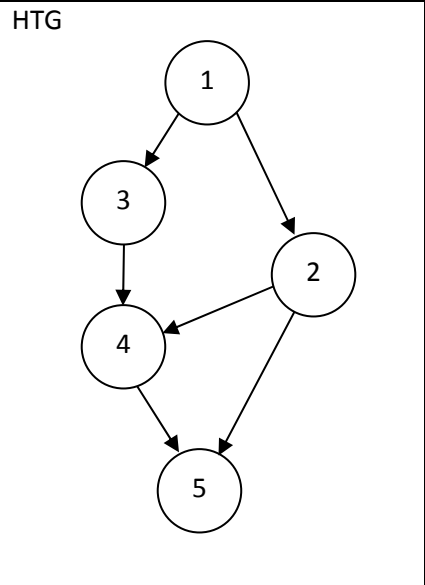
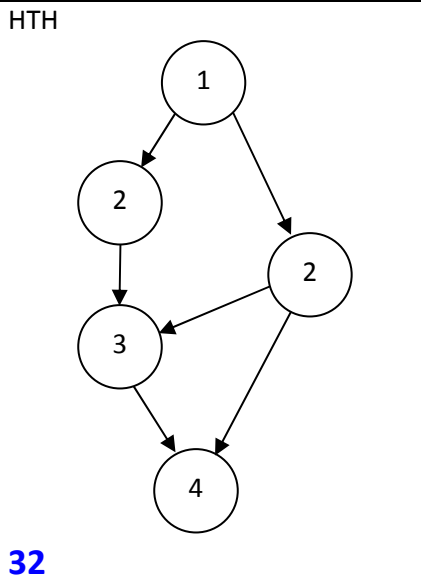
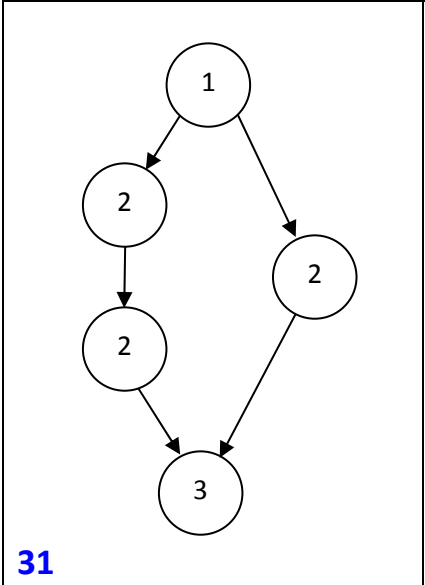
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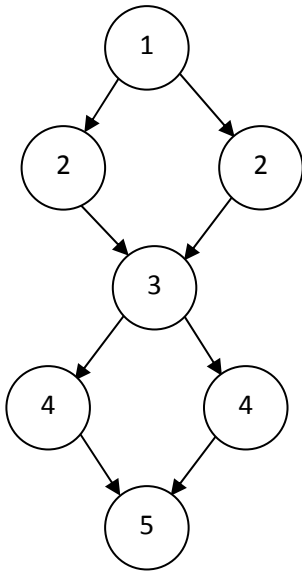


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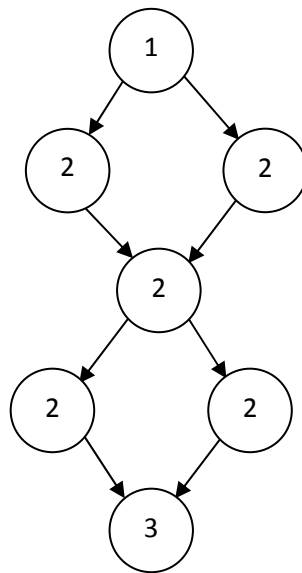




HTH



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